

# Quantum transport in quantum dot cascade structures

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## I. INTRODUCTION

In last years, a significant experimental and theoretical interest has been put into the possibility of development of intraband lasers based on quantum dots [1]–[5]. Due to truly discrete electronic spectrum of quantum dots, most of the undesired scattering and relaxation processes are suppressed, and such devices are expected to have two orders of magnitude lower threshold currents than the corresponding quantum well based devices. However, the lack of precise controlability of quantum dot geometry is probably the main reason why they has not yet been realized. Nevertheless, permanent improvements in technology will certainly at some point make such devices feasible. The physics of such quantum dot devices is essentially different from quantum well devices that possess a continuous spectrum. Polaron effects are known to be important [6] and one can also expect that due to reduced phase space for decoherence, coherent processes are relevant. Therefore, there is an obvious need for the development of an appropriate theoretical framework, capable of treating such processes. In this work, a nonequilibrium Green's function theory of steady-state transport through periodic arrays of single or multiple quantum dots is presented and applied to several simple structures.

## II. THEORY

Firstly, the electronic miniband structure of a quantum dot superlattice is solved using the eight band  $k \cdot p$  method with strain distribution taken into account via continuum elasticity theory, as described in more detail in [7]. Secondly, the wavefunctions obtained are used to construct Wannier states that are localized to a certain period, and present an excellent basis for transport calculations.

Next, Green's functions and self-energies, transformed from time to frequency domain, are represented in Wannier basis. A system of algebraic equations containing Dyson equation, Keldysh relation, and the expressions for self-energies is then formed [8]. In order to close the system of equations, self-energies are approximated using the self-consistent Born approximation (SCBA). The present simulation takes into account electron – longitudinal optical (LO) phonon interaction, as well as the anharmonic decay of LO phonons, which is known to be essential for proper description of relaxation processes in quantum dots [9]. Further development will also

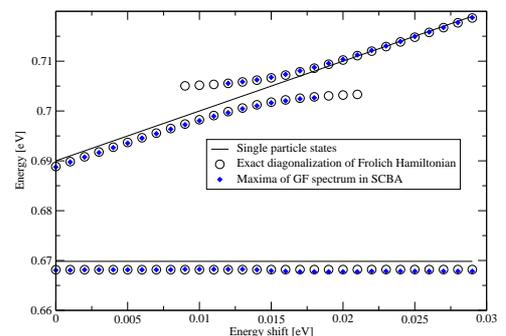


Fig. 1. Comparison of electronic spectrum obtained in the self consistent Born approximation (SCBA) with the one obtained via exact diagonalization of electron-phonon interaction Hamiltonian

include the interaction with acoustic phonons which can be important at low electric fields, when the electronic spectrum still exhibits a miniband structure, and can provide carrier relaxation path within the miniband. They are however not important for the range of electric fields presented here. The system of equations obtained is then solved self-consistently. The localized Wannier basis allows one to employ a tight-binding description where one considers Green's functions and self-energies among the states separated by few periods only. Populations of energy levels and coherences between states, as well as the current through the structure are finally calculated from Green's functions obtained.

## III. VALIDATION OF THE MODEL

The main approximation in the model is the use of SCBA, that is known to be valid in systems with continuous spectrum where polaronic effects are not important, and therefore has been applied in simulations of quantum well based cascade lasers [10]. On the other hand, in the regime of very strong coupling between electrons and phonons, one does not expect this approximation to be valid. In order to validate the SCBA for the InAs/GaAs quantum dot system studied here, electronic spectrum obtained by direct diagonalization of Frölich Hamiltonian of electron phonon interaction was compared with the positions of maxima of the spectral function obtained in SCBA in equilibrium. The results of comparison are shown in Fig. 1, showing that polaronic shifts of the ground state are in good agreement. Additionally, in order to check that

polaronic splitting, that appears when single electron levels are split by an LO phonon energy, is properly taken into account, the excited state energy was artificially varied in that region. The results presented in Fig. 1 clearly indicate that SCBA properly treats polaronic effects in InAs/GaAs quantum dots.

#### IV. RESULTS

The method described has been applied to calculate current density – electric field characteristics of a periodic array of quantum dots. The dot diameter has been taken to be equal to 20 nm, and the height 5 nm, which are typical dimensions of InAs/GaAs quantum dots. The results obtained for several values of the period are presented at Fig. 2. All characteristics exhibit the main peak when LO phonon resonance between the ground state of a certain period and its nearest neighbour. For smaller period lengths, additional peaks appear corresponding to LO phonon resonances with further neighbours. Additionally, a quantification of current density decay with an increase in per

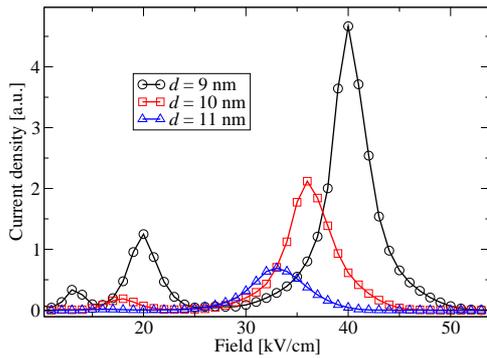


Fig. 2. Current density – electric field characteristics at three values of structure period  $d$ , at a temperature  $T = 300\text{K}$

#### V. CONCLUSION

A model of electron transport in quantum dot arrays, fully capable of treating polaronic and coherent effects, has been developed, validated, and applied to simple quantum dot superlattice structures. It is currently being applied to more complicated structures with the aim of identifying candidate structures for realization of electrically injected intraband quantum dot lasers.

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