

# MICROCAVITY EFFECT ON THE NONLINEAR INTERSUBBAND ABSORPTION IN MULTIPLE- QUANTUM-WELL STRUCTURES: THE STRONG COUPLING REGIME

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## INTRODUCTION

The linear intersubband response of multiple-quantum-well (MQW) structures embedded in microcavities has been studied by many authors for, both fundamental physics and application reasons. The strong-coupling-regime (SCR), in which the dipole coupling between intersubband excitation and the cavity photon gives rise to coherent mixed modes, often referred to as “intersubband cavity polaritons”, has recently received a lot of attention [1-3]. In view of applications for optoelectronic devices, the analysis of the nonlinear intersubband properties is a fundamental and intriguing field of microcavity physics. Previous works devoted to the above mentioned subject concentrated on the systems where conditions for the SCR are not fulfilled [4,5].

In this paper, we discuss theoretically the influence of the cavity effect on the nonlinear intersubband response going beyond the weak coupling regime. We employ a non-iterative numerical method developed in our previous paper [6]. It is based on the transfer matrix method and the so-called sheet model. The advantage of this approach is that it includes, contrary to the approach based on the slowly varying envelope approximation (SVEA) [4] the radiative coupling among QWs. The method is very general and can be used for arbitrary layered structures with

embedded MQWs. Performing numerical calculations we neglect for simplicity the influence of the electron-electron interaction on the intersubband response [7], i.e. we take into account only the saturation effect associated with the light induced redistribution of electrons between the ground and excited subband. The simplified analytical approach based on the mean field approximation is also presented discussed.

## THEORY

Numerical approach. Let us assume that a layered structure with an embedded MQW is sandwiched between the semi-infinite substrate ( $j = 0$ ) and cladding ( $j = m + 1$ ) media. The quasi-two dimensional electron gas (Q2DEG) located in QW is modeled by a 2D sheet [6,7,8]. Light, polarized in the  $x$ - $z$  plane, incidents from the substrate medium (with a dielectric constant  $\varepsilon_0 = \varepsilon_s$ ) at the angle  $\theta$  with respect to the optic axis  $z$  oriented parallel to the growth direction. Then, there is only a single component of the magnetic field  $\mathbf{H}(\mathbf{r}, t) = \mathbf{e}_y H_y(z) e^{i(k_x x - \omega t)}$  in each medium. Let us denote by  $H_{\alpha+}^{(j)}$  and  $H_{\alpha-}^{(j)}$  ( $\alpha = l, u$ ) the complex amplitudes of the magnetic field corresponding to the waves travelling in positive (+) and negative (−)  $z$ -directions, respectively. The subscript  $l(u)$  indicates that we take the amplitude with respect to the plane separating the media  $j$  and  $j + 1$  ( $j - 1$  and  $j$ ).

The reflectance of the structure can be written as

$$R = |H_{l-}^{(0)} / H_{l+}^{(0)}|^2. \quad (1)$$

The relation between the amplitudes of the magnetic field in the  $q$  and  $p (> q + 1)$  media (layers) may be written in terms of the transfer matrix [8]:

$$\begin{bmatrix} H_{l+}^{(q)} \\ H_{l-}^{(q)} \end{bmatrix} = \mathbf{I}_{q,q+1} \prod_{j=q+1}^{p-1} \mathbf{L}_j \mathbf{I}_{j,j+1} \begin{bmatrix} H_{u+}^{(p)} \\ H_{u-}^{(p)} \end{bmatrix}. \quad (2)$$

The explicit expression for matrix  $\mathbf{L}_j$  ( $\mathbf{I}_{i,j}$ ) describing the effect of propagation through the  $j$ th layer ( $i|j$  interface) can be found in our previous paper [8]. The transfer matrix  $\mathbf{I}_{i,j} (\equiv \mathbf{I}_{\mathcal{N}})$  across the 2D sheet can be written in terms of the light intensity dependent 2D intersubband conductivity. In the RWA (and two subband limit) it takes the form [6,7]

$$\sigma_{zz}^{2D}(\omega, |E_z|^2) = \mathcal{A}_{\text{QW}} \frac{1 + i\Delta}{1 + \Delta^2 + |E_z/E_z^{\text{sat}}|^2}, \quad (3)$$

where  $\mathcal{A}_{\text{QW}} = N_s f_{12} e^2 / 2m^* \Gamma_{\text{IT}}$ ,  $m^*$  is the effective electron mass,  $f_{12} = 2m^* \hbar^{-1} \omega_{21} |z_{12}|^2$  is the oscillator strength,  $E_z^{\text{sat}} = \hbar(\Gamma_{\text{IT}} \Gamma_{21})^{1/2} / |d_{12}|$ ,  $\Delta = (\omega - \omega_{21}) / \Gamma_{\text{IT}}$ ,  $\tau_{\text{IT}} = 1 / \Gamma_{\text{IT}}$  is the dephasing time,  $\tau_{21} = 1 / \Gamma_{21}$  is the phenomenological life time,  $E_z$  is the normal component of the electric field,  $N_s$  is the surface

electron density, and  $\omega_{21}$  ( $d_{12} = ez_{12}$ ) is the intersubband resonant frequency (the dipole matrix element).

The reflectance of the microcavity (at a fixed frequency  $\omega$  or fixed intensity  $I^{\text{in}}$  of the incident radiation in the substrate medium) has been calculated numerically using the recursion method [6]. More precisely,  $H_{l+}^{(0)}$  (and  $H_{l-}^{(0)}$ ) corresponding to an arbitrary value of  $H_{u+}^{(m+1)}$  (and  $\omega$ ) has been obtained moving from the output side (medium  $m + 1$ ) to the input side (medium 0). Note that we employ the fact that the amplitude  $H_{l+}^{(0)}$  can be treated as a single value function of  $H_{u+}^{(m+1)}$  which can be taken as a (real) parameter.

Analytical approach. To get useful analytical results retaining main qualitative features of the system we employ an approach similar to the one developed in [9]. Let us consider the following simple microcavity-MQW system. The MQW (composed of  $N_{\text{QW}}$  wells) is located between the front (at  $z = 0$ ) and back (at  $z = L_{\text{cav}}$ ) mirrors with reflection coefficients  $r_f e^{-i\psi_f}$  and  $e^{-i\psi_b}$ , respectively. The half-space  $z < 0$  occupies the substrate material. The difference between the dielectric constant of the substrate material ( $\varepsilon_s$ ), the barrier material ( $\varepsilon_b$ ) and the well material ( $\varepsilon_w$ ) is neglected ( $\varepsilon_s = \varepsilon_b = \varepsilon_w$ ). We also assume that  $T_f = (1 - r_f^2) \ll 1$ . In this limit the

resonant frequency ( $\omega_{\text{cav}}$ ) and the half line width ( $\Gamma_{\text{cav}}$ ) of the ground mode of the passive cavity are given by

$$\omega_{\text{cav}} = \frac{(2\pi - \psi_f - \psi_b)c}{2\varepsilon_w^{1/2} L_{\text{cav}} \cos \theta}, \quad \Gamma_{\text{cav}} = \frac{T_f c}{4\varepsilon_w^{1/2} L_{\text{cav}} \cos \theta}, \quad (4)$$

We describe the influence of the intersubband excitations on the cavity reflection spectra within the mean field approximation [9]. This approximation is equivalent to the following substitution:

$$|E_z/E_z^{\text{sat}}|^2 \rightarrow X = L_{\text{cav}}^{-1} \int_0^{L_{\text{cav}}} |E_z/E_z^{\text{sat}}|^2 dz. \quad (5)$$

In this limit the reflectance ( $r^{2\text{D}}$ ) and transmittance ( $t^{2\text{D}}$ ) of the sheet does not depend on its position in the microcavity and can be written as

$$r^{2\text{D}} = \Lambda/(1 + \Lambda), \quad t^{2\text{D}} = 1/(1 + \Lambda), \quad (6)$$

where  $\Lambda = \bar{\Lambda}(1 + i\Delta)/F$ ,  $F = 1 + \Delta^2 + X$  and

$$\bar{\Lambda} = (4\pi/c\sqrt{\varepsilon_w})\mathcal{A}_{\text{QW}} \tan \theta \sin \theta. \quad (7)$$

Since, for realistic values of QW parameters,  $|\Lambda| \ll 1$  the relations (6) can be approximated by  $r^{2\text{D}} = 0$  and  $t^{2\text{D}} = \exp(-\Lambda)$ . (This approximation is consistent with the SVEA.) To proceed further we assume that  $\Lambda_{\text{MQW}} = \bar{\Lambda}N_{\text{QW}}$  and  $T$  go to zero, while their ratio  $C = \Lambda_{\text{MQW}}/T$  remains constant and arbitrarily large.

Taking into account the above mentioned simplifications we can get the relation between  $Y$  and  $X$  by summing, like in Ref. [9], the bi-directional multireflected (from the mirrors) waves and averaging over the cavity width  $L_{\text{cav}}$ . The result is

$$\frac{Y}{X} = \left| \left( 1 + \frac{2C}{F} \right) + i \left( \frac{2C\Delta}{F} - \frac{\omega - \omega_{\text{cav}}}{\Gamma_{\text{cav}}} \right) \right|^2, \quad (8)$$

where  $Y = 8 \sin^2 \theta I^{\text{in}} \mathcal{F} / I_{\text{sat}} T$ ,  $I_{\text{sat}} = c \sqrt{\epsilon \omega} (E_z^{\text{sat}})^2 / 8\pi$  is the saturation intensity and  $\mathcal{F}$  it is a factor (of the order unity) depending on  $\psi_b$  and  $\psi_f$ . In the case of metallic mirrors one can take  $\mathcal{F} \cong 1$ . (The above equation has the same form as the one discussed in Ref. [10]. Note that the authors of this paper considered an optical cavity filled with two-level atoms starting from the coupled Maxwell-Bloch equations.) In the resonant case ( $\omega - \omega_{21} = \omega - \omega_{\text{cav}} \equiv \Omega$ ) we get [10]

$$\frac{X}{Y} = \left| \frac{\Gamma_{\text{cav}}(\Omega + i\Gamma_{\text{IT}})}{(\Omega + i\Gamma_{\text{cav}})(\Omega + i\Gamma_{\text{IT}}) - \tilde{\Omega}_{\text{VR}}^2} \right|^2 \quad (9)$$

$$= \left| \frac{A_+}{\Omega - \Omega_+} + \frac{A_-}{\Omega - \Omega_-} \right|^2,$$

with

$$A_{\pm} = \Gamma_{\text{cav}}(\Gamma_{\text{IT}} + \Omega_{\pm}) / (\Omega_{\mp} - \Omega_{\pm}) \quad (10)$$

$$\Omega_{\pm} = -i \frac{\Gamma_{IT} + \Gamma_{cav}}{2} \pm \sqrt{\tilde{\Omega}_{VR}^2 - \left( \frac{\Gamma_{IT} - \Gamma_{cav}}{2} \right)^2}, \quad (11)$$

$$\tilde{\Omega}_{VR}^2 = \Omega_{VR}^2 \left[ 1 + X \Gamma_{IT}^2 / (\Omega^2 + \Gamma_{IT}^2) \right]^{-1}, \quad (12)$$

$$\Omega_{VR}^2 = \frac{\pi f_{21} e^2 \mathcal{N} \tan^2 \theta}{m^* \epsilon_w} = \frac{2\pi \omega_{21} |d_{21}|^2 \mathcal{N} \tan^2 \theta}{\epsilon_w \hbar}, \quad (13)$$

where  $\mathcal{N} = N_{QW} N_s / L$  is the MQW electron density. Thus we can see that in the limit of low excitations,  $X \ll 1$ , the system behaves like two coupled harmonic oscillators corresponding to the normal mode ( $\Omega_{\pm}$ ) of the MQW-microcavity system. When  $\Gamma_{IT} = \Gamma_{cav}$ , the minimal separation between the above modes is of twice of the vacuum Rabi frequency  $\Omega_{VR}$ .

The inspection of the numerical results reported in [10] indicates that, when the intensity increases the peaks of the function  $X(\Omega)$  move towards the on-resonance centre and deform into a multivalued shape before they meet. In the approximation used here the power ( $P$ ) dissipated in the system is proportional to the product  $X \operatorname{Re} \sigma_{zz}^{2D} \propto X/F$ . Since  $F$  depends on  $\Omega$  much weakly than  $X$  the spectral shapes of the absorptance of the structure  $A(= 1 - R)$  and  $X$  should be qualitatively similar, particularly in the region where  $\Delta^2 \gtrsim X$ .



## RESULTS AND DISCUSSION

The numerical calculations have been done for the resonant asymmetric [metal clad (Fig. 1) and purely dielectric (Fig. 2)] microcavities similar to those studied recently in Refs. [1] and [2], respectively. The obtained results are displayed in Figs. 3 and 4. As one can expect we observe quantitatively an anharmonic evolution from the two vacuum Rabi dips in the low-intensity regime to the single reflection dip in the high intensity limit. In the transition region, the vacuum Rabi dips shift and deform. Reflection exhibits then multistable behavior. The above mentioned evolution is consistent with the one predicted by the simplified theoretical description based on the mean field approximation [9,10]. We have checked that in the case of the structure considered in Fig. 3 the reflection dip evolution is substantially modified by: (i) the Drude absorption in Au and n-GaAs layers, and (ii) the formation of higher order polaritons (a small central dip).

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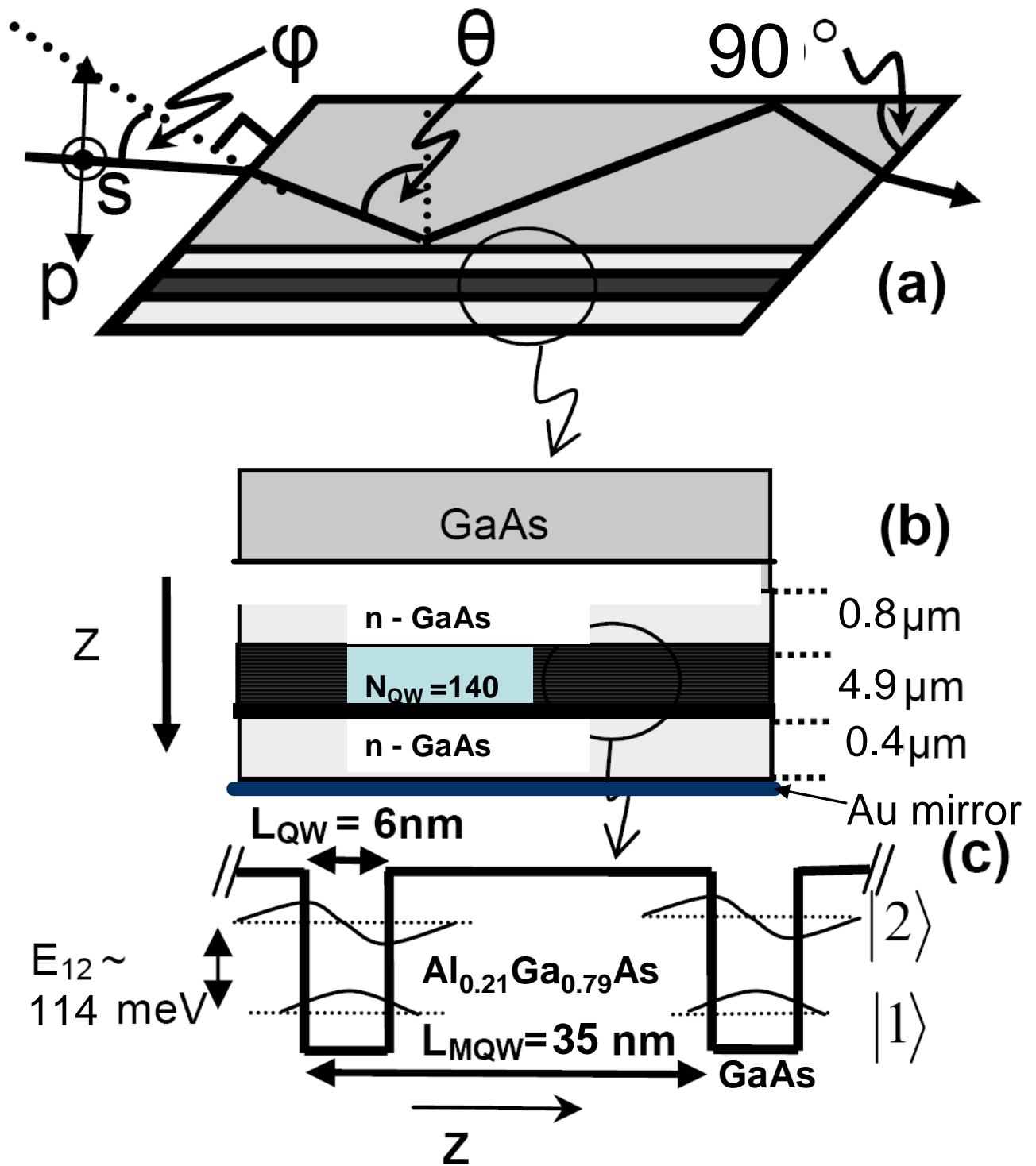


Figure 1: The prism coupling scheme (a) and the details of the metal clad microcavity structure studied in Ref. [1] (b,c).

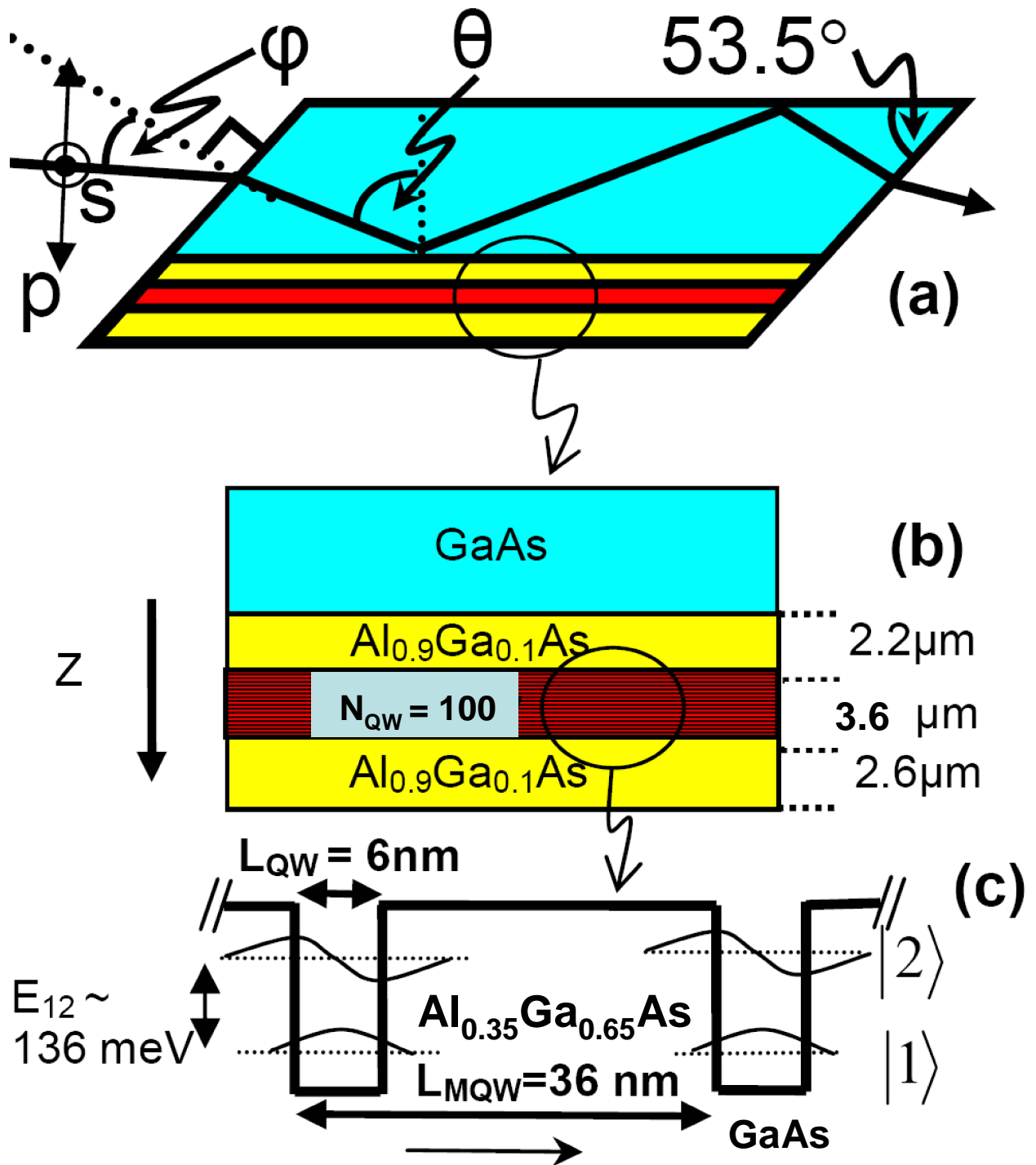


Figure 2: The prism coupling scheme (a) and the details of the dielectric microcavity similar to studied in Ref. [2] (b,c).

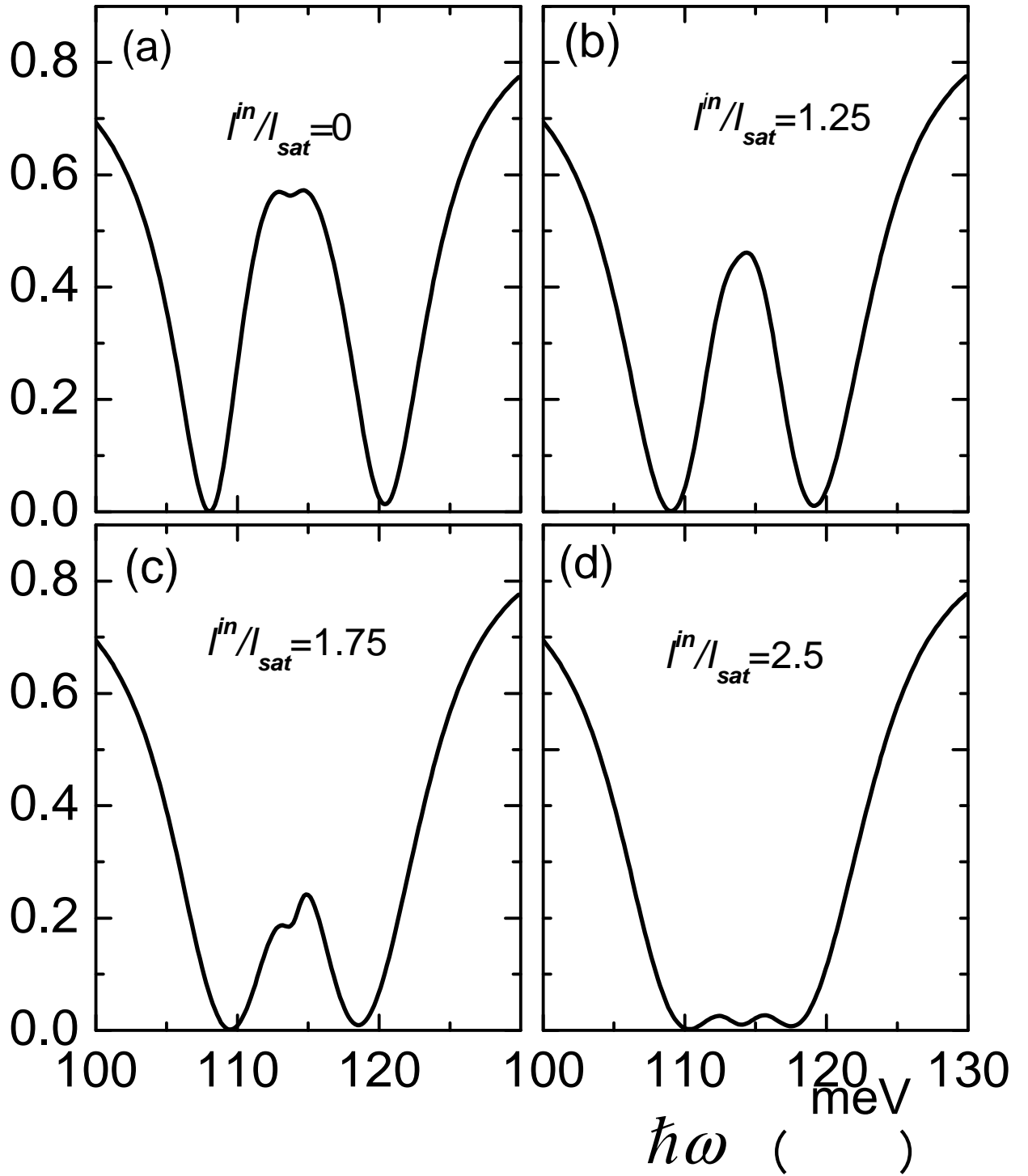


Figure 3: The spectral shape of  $R$  calculated for the structure presented in Fig.1 at different values of  $I^{\text{in}}/I_{\text{sat}}$ .  $N_s \times f_{12} = 2 \times 10^{10} \text{ cm}^{-2}$ ,  $\hbar\Gamma = 1.1 \text{ meV}$ ,  $\theta = \theta_{\text{res}} = 73.2^\circ$ .

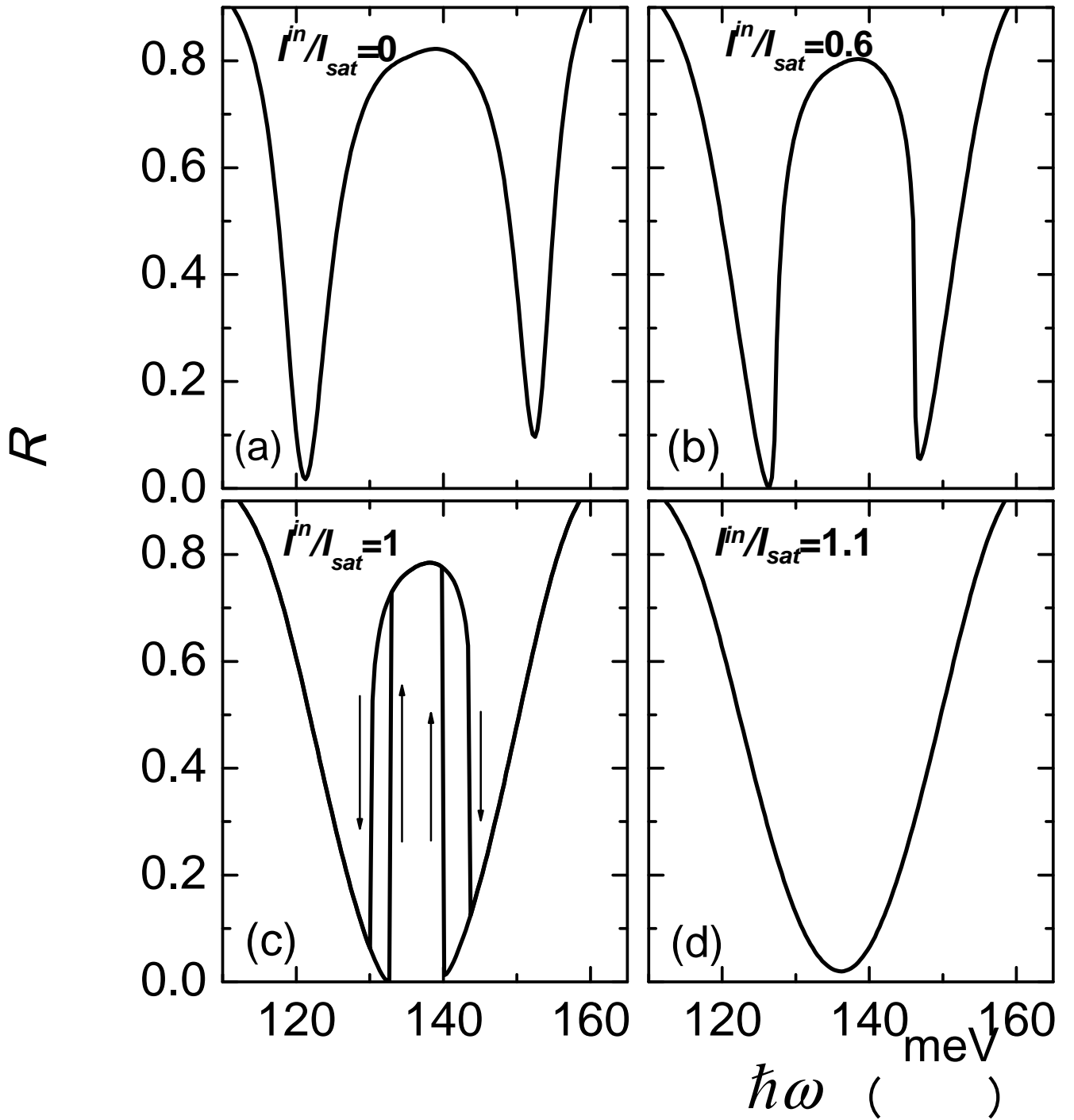


Figure 4: The spectral shape of  $R$  calculated for the structure presented in Fig. 2 at different values of  $I^{in}/I_{sat}$ .  $N_s \times f_{12} = 10^{11} \text{ cm}^{-2}$ ,  $\hbar\Gamma = 3.8 \text{ meV}$ ,  $\theta = \theta_{res} = 67.91^\circ$ .