

Directivity of sub-wavelength wire lasers

E.E. Orlova

Institute for Physics of Microstructures RAS

Acknowledgements:

Experimental investigation of the far field of THz cascade lasers:

**T.O. Klaassen, J.N. Hovenier, A.J. Adam, I. Kašalynas,
J.R. Gao**

Kavli institute of Nanosciences, TU Delft, The Netherlands

THz cascade lasers:

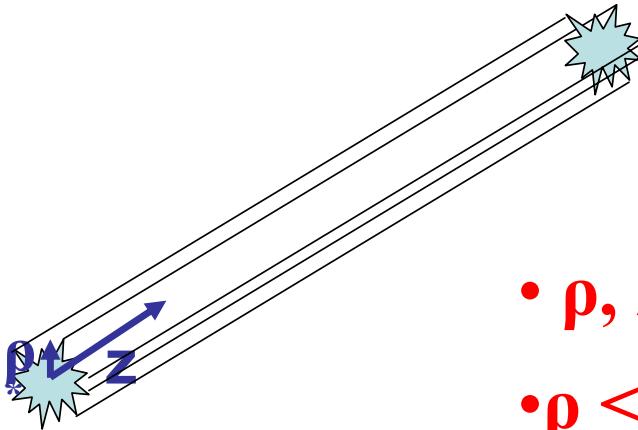
B.S. Williams, S. Kumar, Q. Hu

MIT, Cambridge, Massachusetts, USA

J. L. Reno

Sandia National Laboratories, Albuquerque, USA

Wire lasers



- $\rho, \lambda \ll L$;
- $\rho < \lambda$
- **uniform or periodic along z direction with period $\ll \lambda$;**

**High directivity laser with
subwavelength aperture.**

Is it possible?

Methods used to improve directivity of lasers with small transverse dimensions

- External optics (lenses, horn antennas)
- Emission is coupled out through the substrate
- Emission from the surface parallel to active layer (grating on top of the structure)

Effective aperture is enlarged

Zh. I. Alferov, V. M. Andreev, S. A. Gurevich R. F., Kazarinov, V. R. Larionov, M. N. Mizerov, and E. L., Portnoy, IEEE J. Quant. Electron. QE-11, 449 (1975).

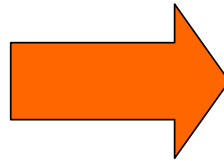
- A set of lasers as a phased array

High directivity laser with subwavelength aperture - what for?

Small aperture

+

Low divergency



Local laser excitation for applications in biology, medicine, effective coupling of THz QC lasers, lasers with higher frequencies - applications in optical memory, quantum information etc.



С КАЖДЫМ ДНЕМ ВСЕ РАДОСТНЕЕ ЖИТЬ!

“Every day more joy to live!”

Local laser excitation for applications in biology, medicine, effective coupling of THz QC lasers, lasers with higher frequencies - applications in optical memory, quantum information etc.

However!

Diffraction limit:

$$a > \lambda$$

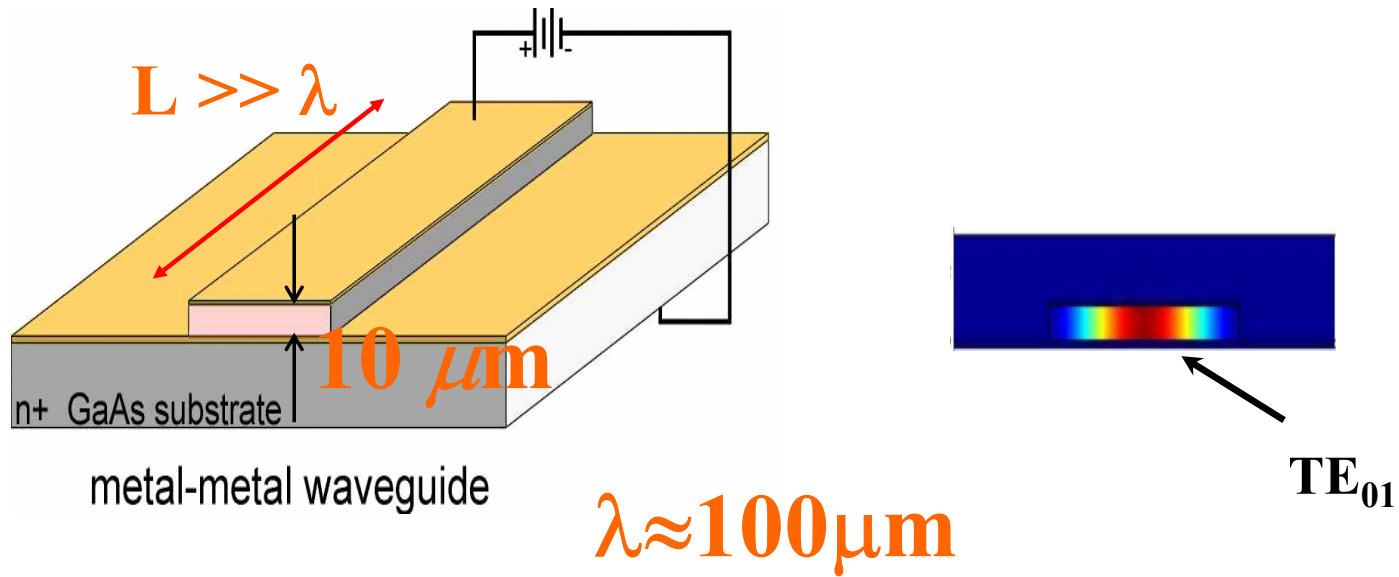
$$\sin\theta \approx \lambda/a$$

wavelength

Aperture size

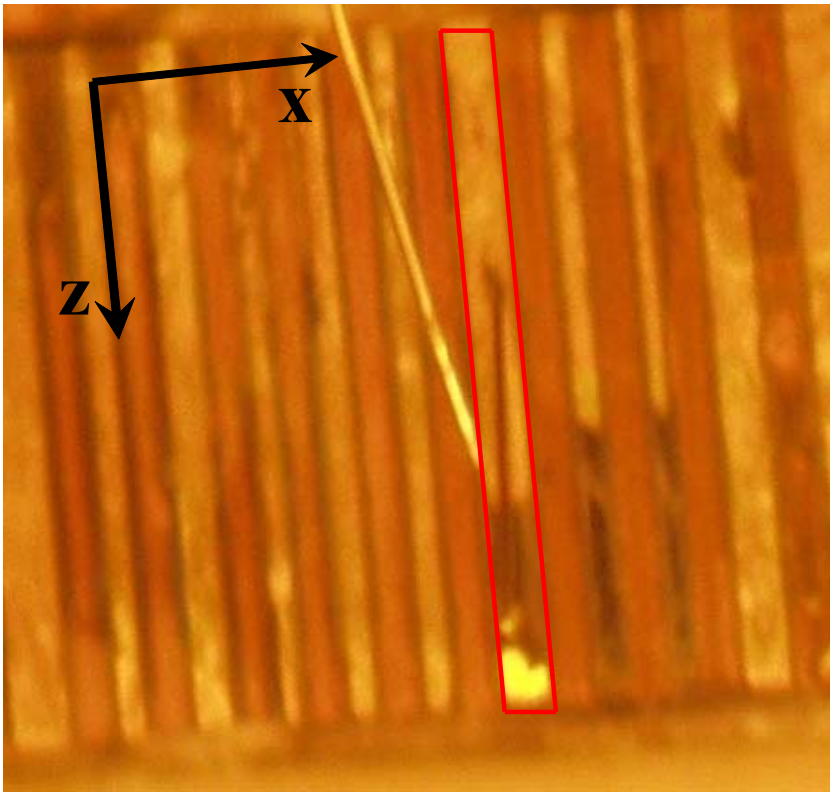
What happens when $a < \lambda$?

Waveguide of the THz quantum cascade laser



MIT, Sandia, 2003

Structure view

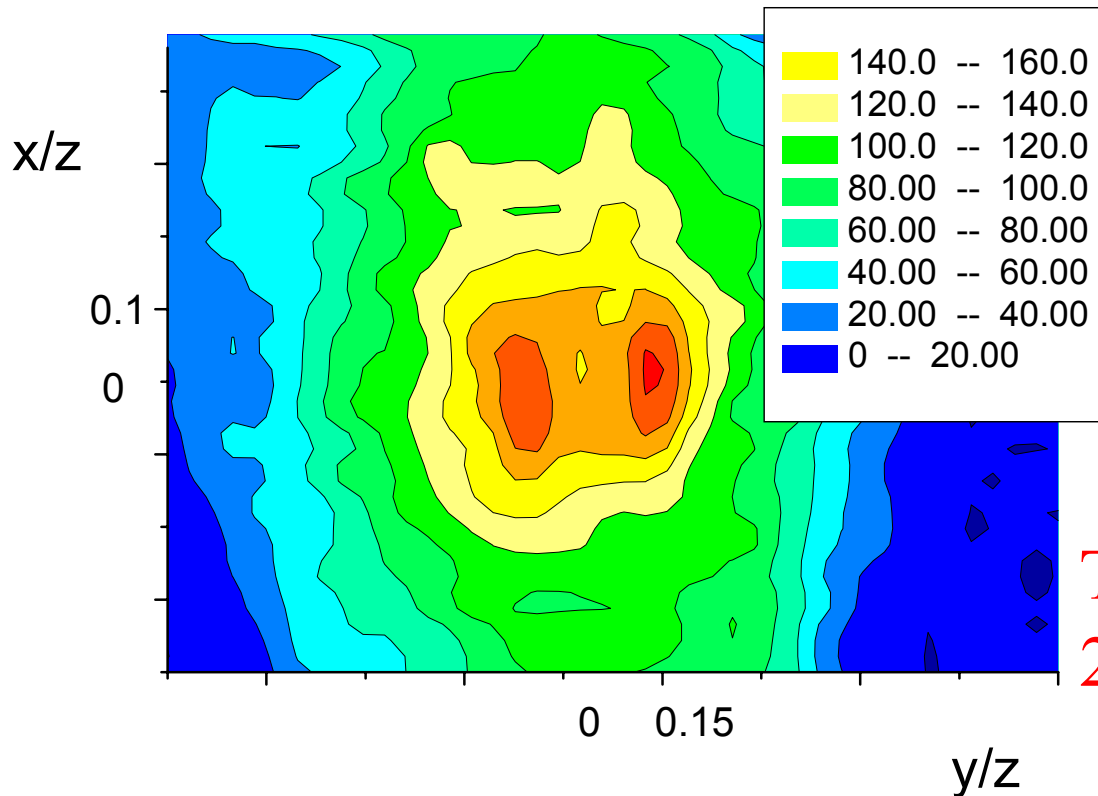


z: $L=1180\mu\text{m}$.

x: $a=40\mu\text{m}$

y: $h=10\mu\text{m}$

First result: narrow maximum in the far field of quantum cascade laser
1180/40/10 μm , $\lambda \approx 100 \mu\text{m}$, $\theta < 0.1 \pi$



TU Delft
2004

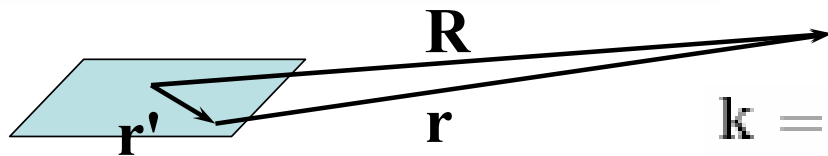
Equivalent currents approach:

Far field is expressed in terms of equivalent current produced by the laser mode within the laser medium with $\epsilon \neq 1$:

$$\mathbf{J}_{eq} = \mathbf{J}_e + \frac{j\omega(\epsilon - 1)}{4\pi} \mathbf{E}_i$$

$$\mathbf{A} = \frac{1}{c} \int_V \mathbf{J}_{eq} \frac{\exp(-jkr)}{r} dV,$$

$$\mathbf{E} = -\frac{j\omega}{c} \mathbf{A} + \frac{c}{j\omega} \nabla(\nabla \cdot \mathbf{A})$$



Far Field Zone:

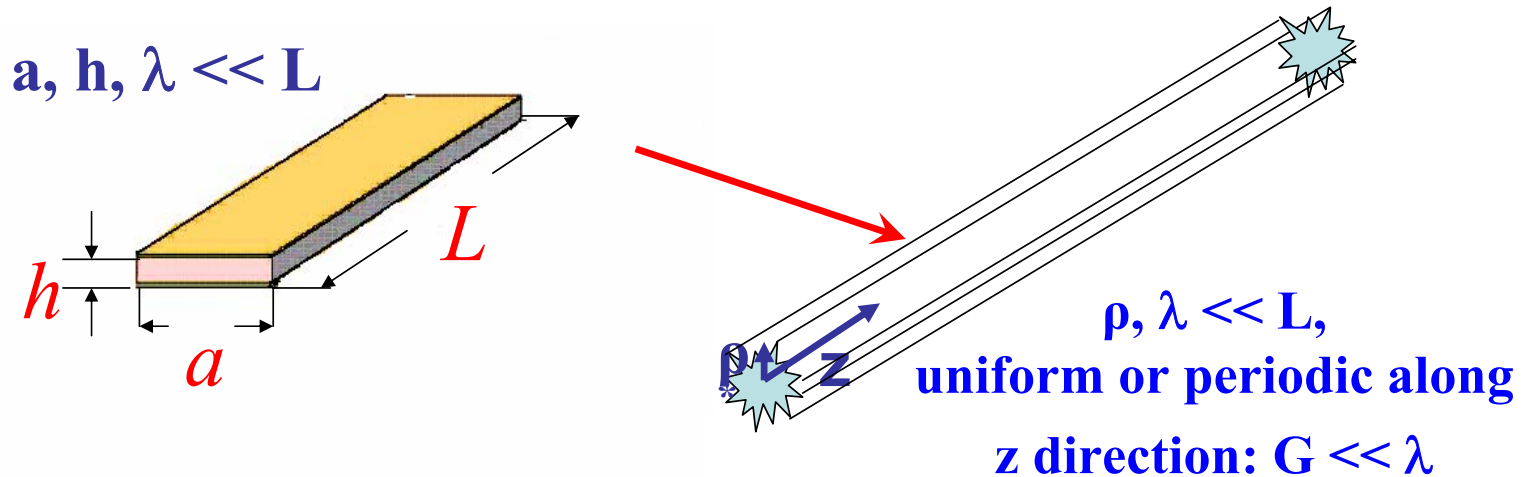
$$\mathbf{k} = k\mathbf{R}/R \quad kr \simeq kR - k\mathbf{r}'$$

$$\mathbf{E} = -\frac{j\omega}{c^2} \frac{\exp(-jkR)}{R} \sum_i \mathbf{E}_i \int_V J_{eq}^i \exp(jk\mathbf{r}') d\mathbf{r}'$$

\mathbf{E}_i – polarization factor

$$\mathbf{E}_i = \mathbf{x}_0^i - \mathbf{r}_0(\mathbf{x}_0^i \mathbf{r}_0) \quad \mathbf{r}_0 = \mathbf{R}/R$$

Wire model of laser mode structure



Exact solution for $L \rightarrow \infty$ - Bloch functions:

$$J_{eq}^i = \tilde{J}^i(\rho, z) (\exp(jqz) \pm \exp(-jqz))$$

Boundary
conditions at
 $|z|=L/2$



Discrete spectrum of ω



Edge effects are neglected
($L \gg a, L \gg \lambda$)

Far field of wire laser

$$J_{eq}^i = \tilde{J}^i(\rho, z) (\exp(jqz) \pm \exp(-jqz))$$

$$\mathbf{E} = -\frac{j\omega \exp(-jkR)}{c^2 R} \sum_i \mathbf{E}_i \int_V J_{eq}^i \exp(j\mathbf{k}\mathbf{r}') d\mathbf{r}'$$

Transverse and longitudinal integrals factor out

$$\mathbf{E} = -\frac{j\omega \exp(-jkR)}{c^2 R} L \sum_i \mathbf{E}_i F_i^\perp F_i^z$$

Transverse factor

$$F_i^\perp = \int_S \tilde{J}^i(\rho) \exp(j\mathbf{k}_\perp \rho) d\rho$$

Longitudinal factor

$$F_i^z = \int \exp(jk_z z) (\exp(jqz) \pm \exp(-jqz)) dz$$

Transverse factor

This term is analogous to that of aperture methods of far field calculation

$$F_i^\perp = \int_S \bar{J}^i(\rho) \exp(j\mathbf{k}_\perp \rho) d\rho$$

$$\mathbf{a} > \lambda$$

$$\Theta \simeq \lambda/a$$

$$\mathbf{a} < \lambda$$

Exponential term is expressed as a series of a/λ
→ weak angular dependence

Longitudinal factor

φ - phase difference of radiation from sources along the laser

$$F_i^z = \frac{\sin(\varphi_+)}{\varphi_+} \mp \frac{\sin(\varphi_-)}{\varphi_-}$$

$$\varphi_{\pm} = (q \pm k_x)L/2$$

$$\varphi_{\pm} = (c/c_L \pm \cos(\theta))\pi L/\lambda$$

maxima: $|\varphi_{\pm}| = |(c/c_L \pm \cos(\theta))\pi L/\lambda| = 0, (n + 1/2)\pi$

q- longitudinal wave vector of the mode

q > k (c > c_L)

q = k (c = c_L)

Cone-like maxima,

$$\cos(\theta + d\theta) - \cos(\theta) = \lambda/L$$

Narrow beam along the longitudinal axis

$$\Theta \simeq \sqrt{\lambda/L}$$

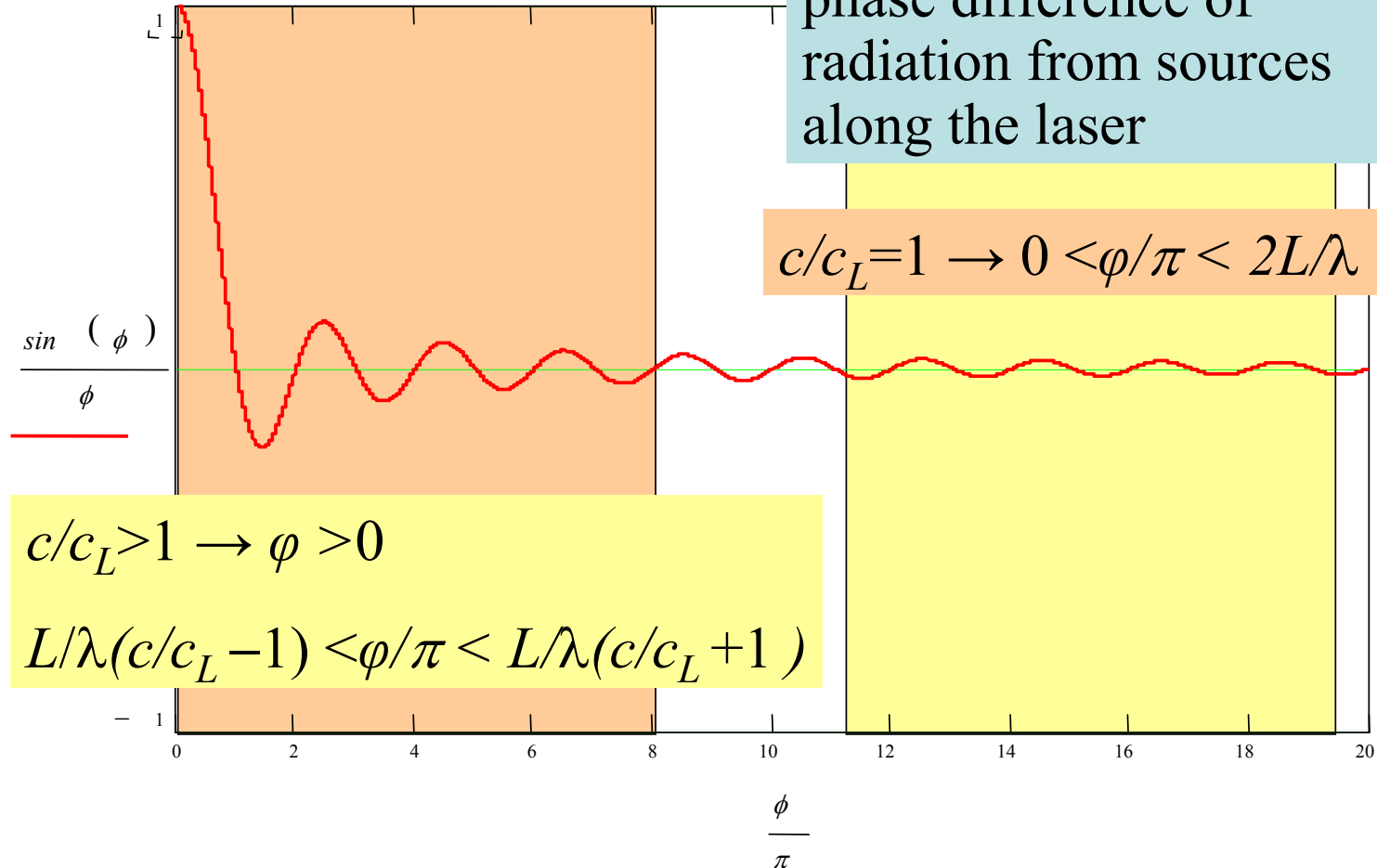
phase difference is nonzero in all directions

$\Theta = 0$ direction of zero phase difference

Sin(φ)/ φ

$$\varphi = \pi L/\lambda (c/c_L - \cos(\theta))$$

phase difference of radiation from sources along the laser



$$c/c_L = 1 \rightarrow 0 < \varphi/\pi < 2L/\lambda$$

$$c/c_L > 1 \rightarrow \varphi > 0$$

$$L/\lambda(c/c_L - 1) < \varphi/\pi < L/\lambda(c/c_L + 1)$$

Synchronous modes – narrow beam:

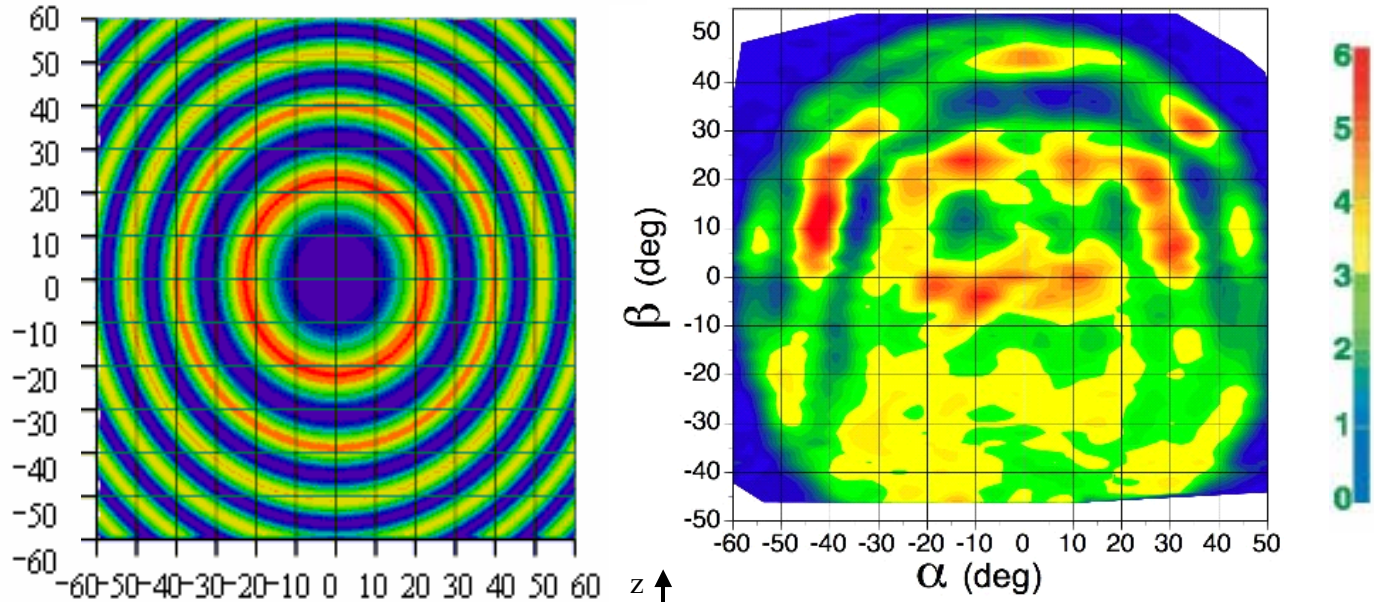
$$L \gg a, L \gg \lambda, a < \lambda, q = k \text{ (} c=c_L \text{)}$$

$$\Theta \simeq \sqrt{\lambda/L}$$

When $N_F = a^2/2\lambda L \ll 1$

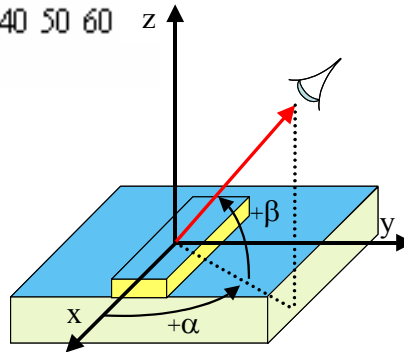
$$\Theta \ll \lambda/a$$

Theory/experiment for $q > k$

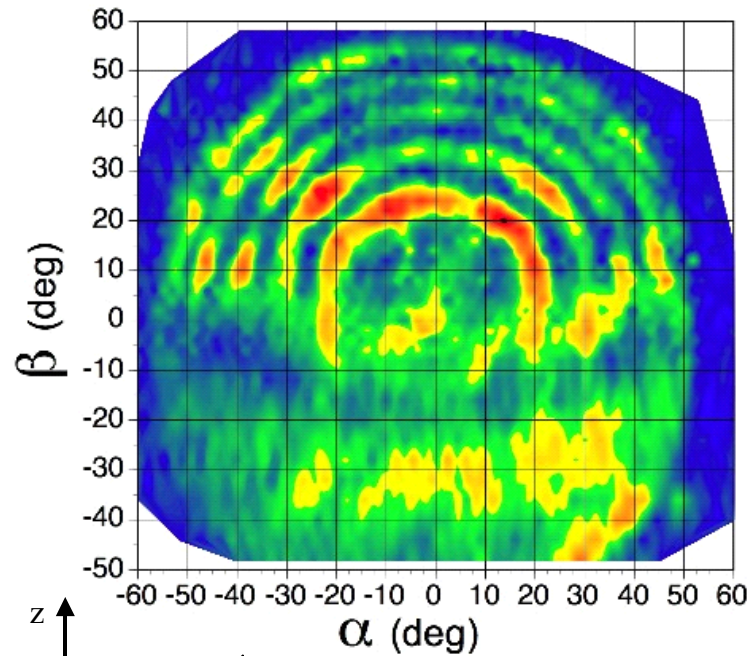
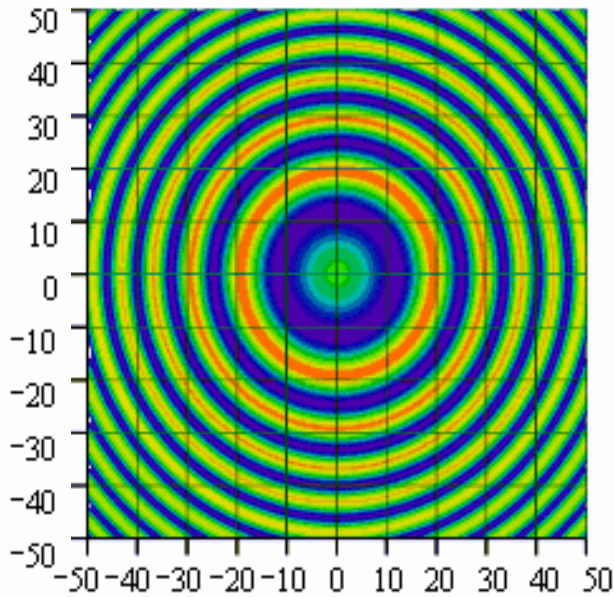


$L=670 \mu\text{m}$,

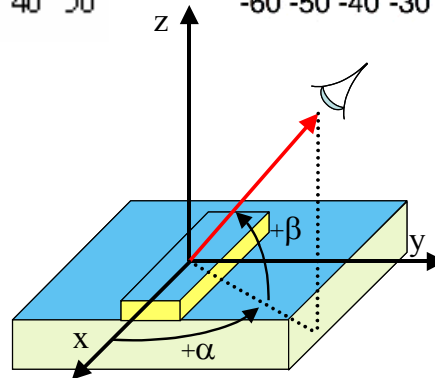
$\lambda = 102.7 \mu\text{m}$



Theory/experiment for $q > k$

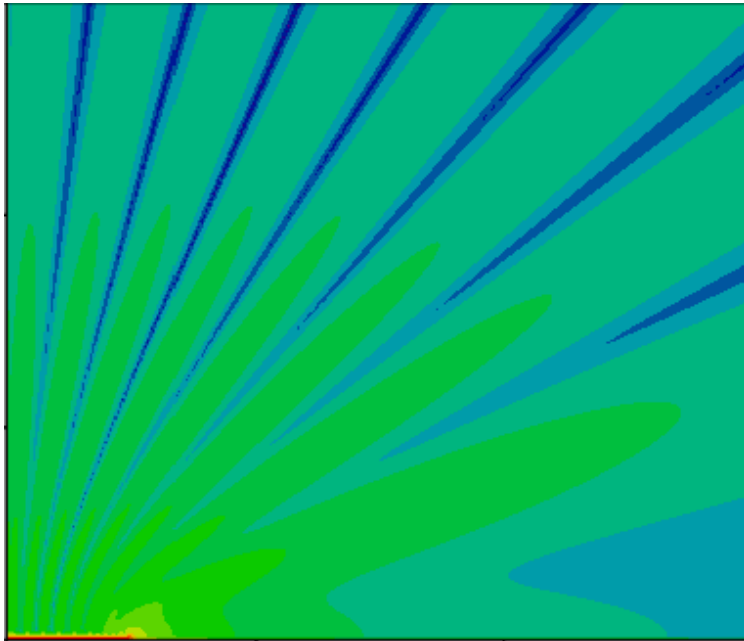


$L=1500 \mu\text{m}$,
 $\lambda = 109.1 \mu\text{m}$

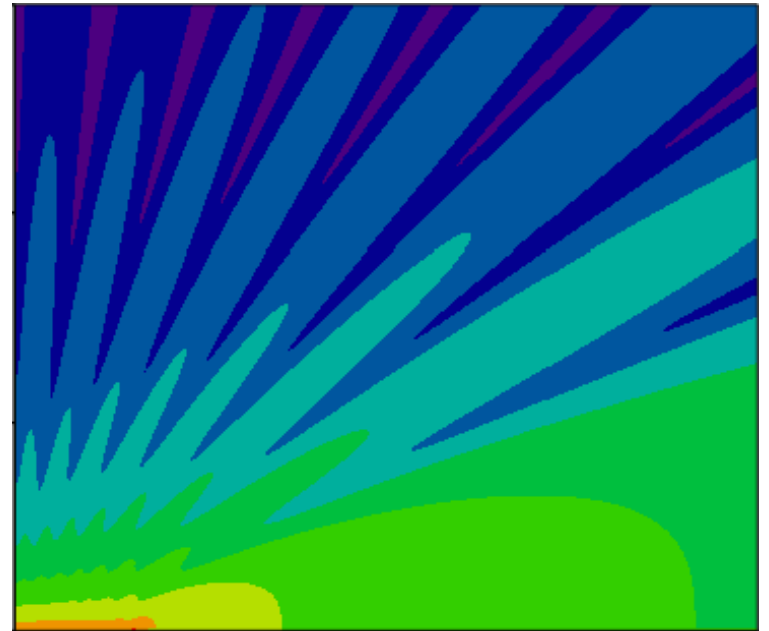


Far and near field
of infinitely thin wire laser

$q > k$ “cabbage”



$q = k$: “flower bulb”



Conditions of realization for synchronized modes $q = k$ ($c=c_L$)

~~Grating with period $2\pi/k$?~~

Dispersion relation inside laser medium: $q^2 = k^2 n^2 - k_{\perp}^2$

Longitudinal wave vector can be changed by changing transverse parameters of the structure.

Properties of synchronized modes $q = k$ ($c=c_L$)

Dispersion relation: $k_{\perp}^2 = q^2 - k^2 = 0$

- no exponential decay outside laser medium

Transverse field distribution is described by quasi-static equation: $\Delta_{\perp} \Pi = 0$

Power decay is determined by system geometry:

Cylinder monopole –

$E \sim 1/\rho \rightarrow \rho(P=P_t/2) = \infty$ effective aperture for infinite wire is infinite

Cylinder dipole –

$E \sim 1/\rho^2 \rightarrow \rho(P=P_t/2) = \sqrt{2}a$ effective aperture is subwavelength

Conclusions:

- Far field of subwavelength wire lasers

$$L \gg a \quad L \gg \lambda \quad a < \lambda$$

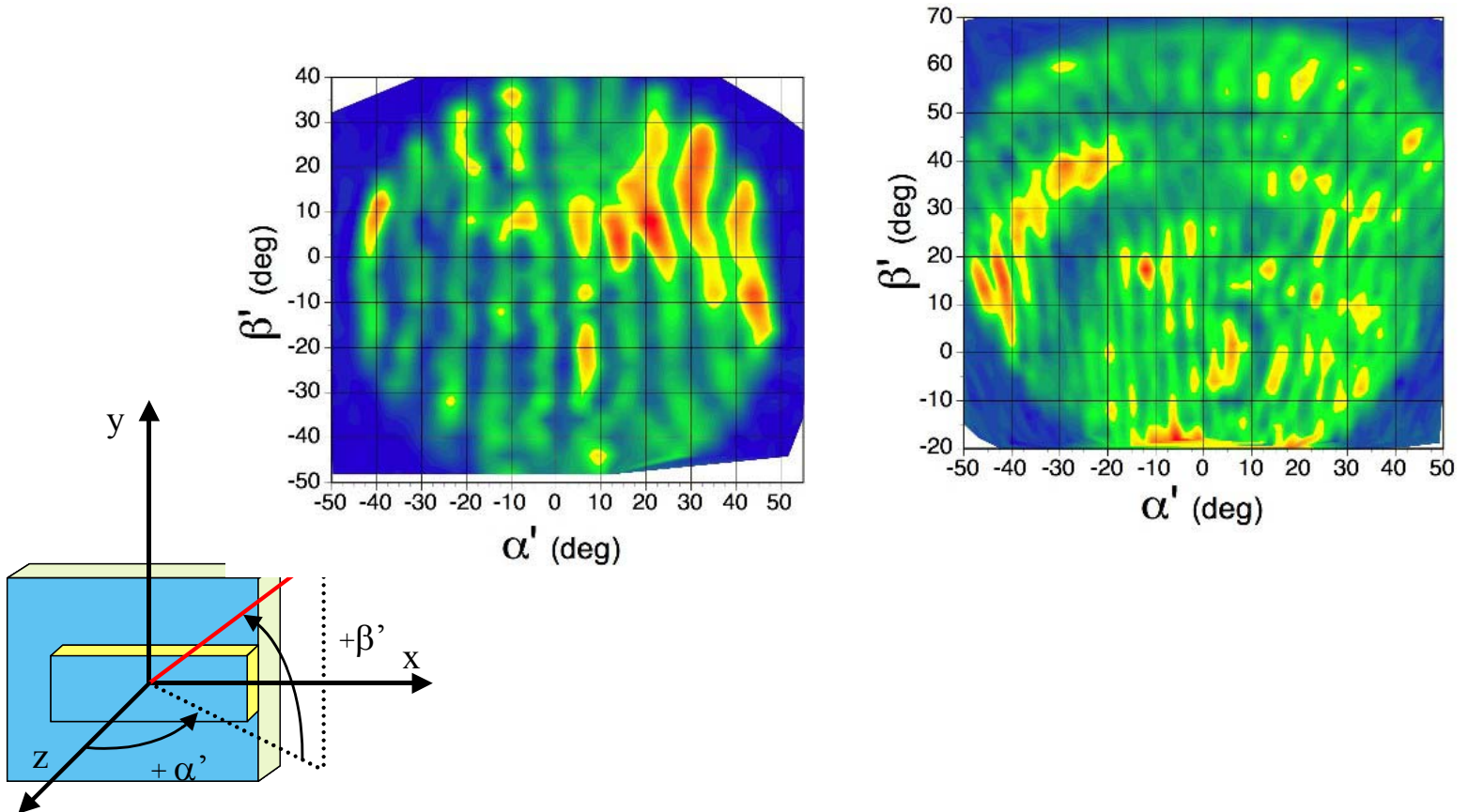
is determined by interference of radiation from longitudinal mode distribution

- High directivity can be achieved from the modes of subwavelength laser at synchronism of longitudinal phase velocity with that of light with beam width :

$$\Theta \simeq \sqrt{\lambda/L}$$

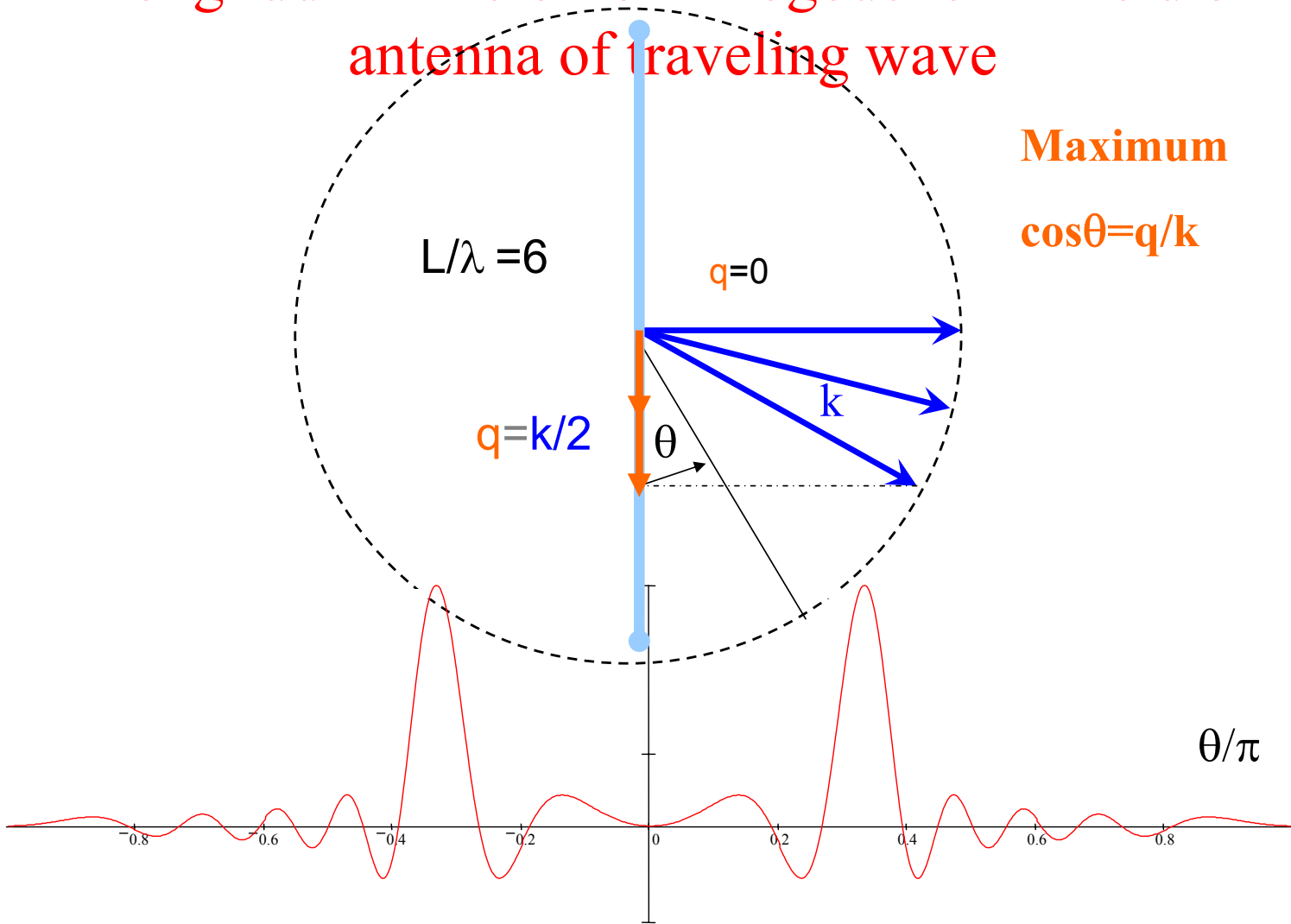
- Synchronous modes have relatively low confinement; realization of narrow beam emission from synchronous modes requires mode selection for single mode generation

Emission in the plane parallel to the axis

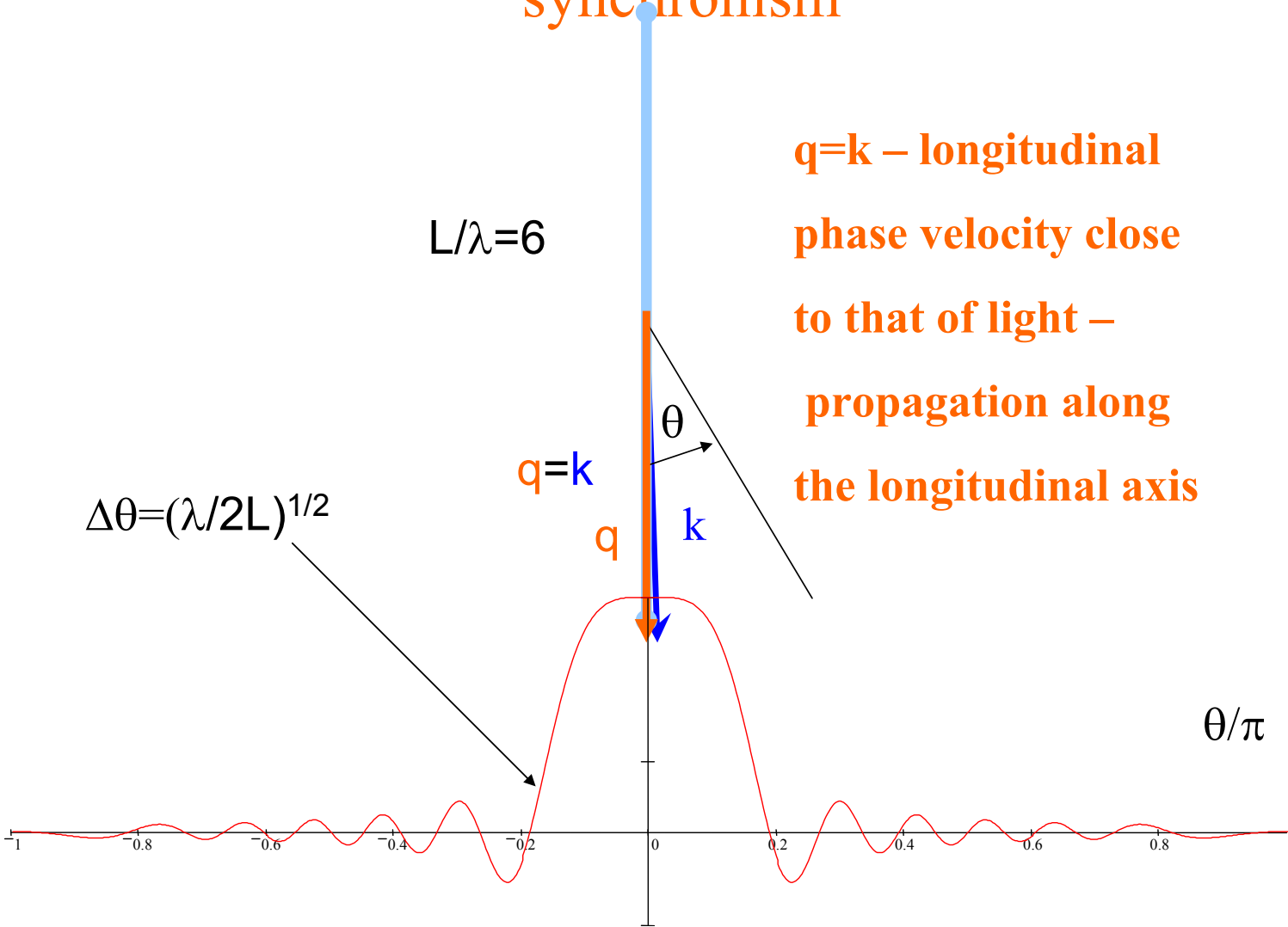


Longitudinal factor is analogous to far field of antenna of traveling wave

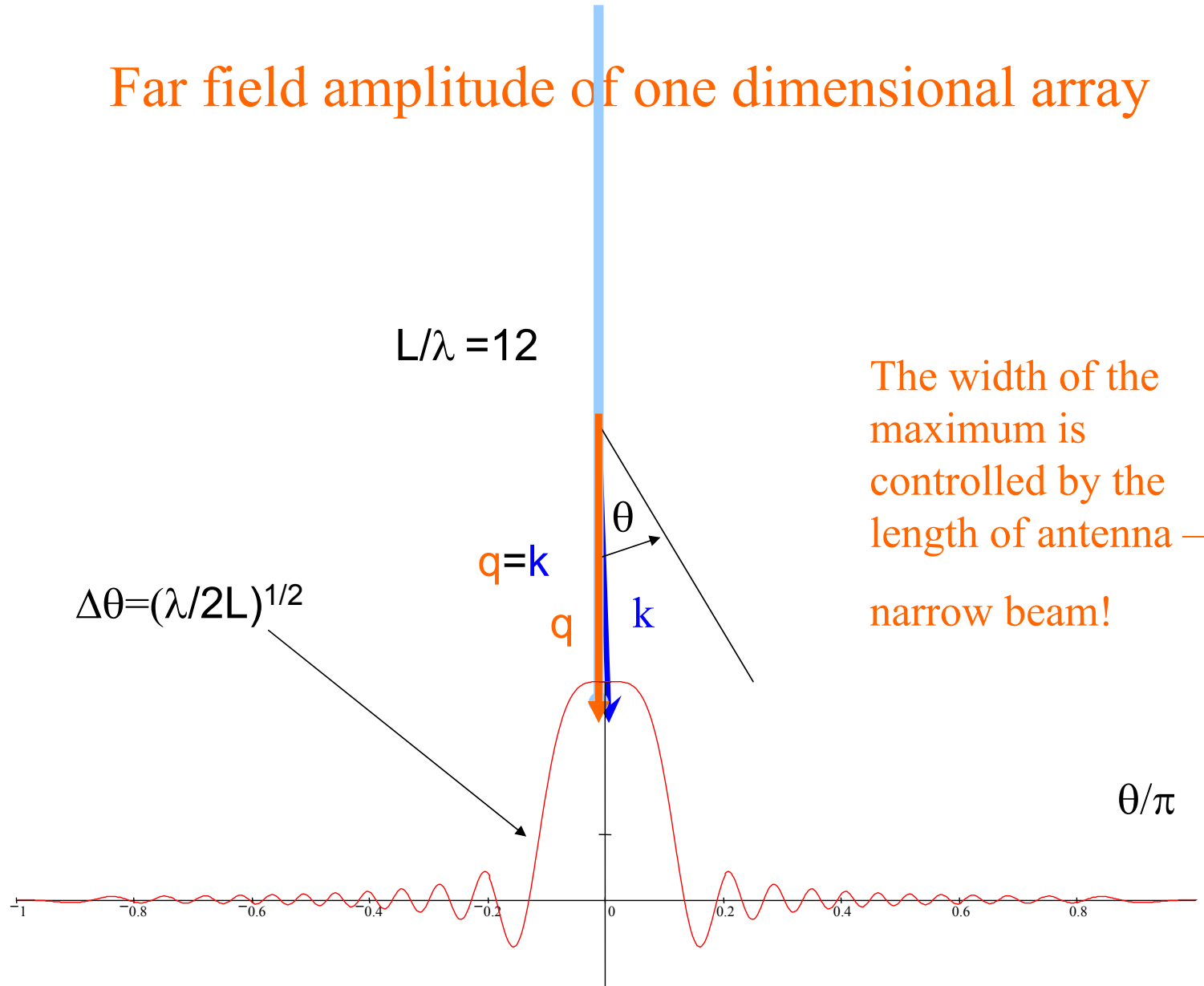
Maximum
 $\cos\theta = q/k$



Far field of travelling wave antenna at phase synchronism



Far field amplitude of one dimensional array



Far field amplitude of one dimensional array

